

Motion Control and Dynamic Load Carrying Capacity of Mobile Robot via Nonlinear Optimal Feedback

M.H. Korayem¹, M. Irani² and S. Rafee Nekoo³

Robotic Research Laboratory, Center of Excellence in Experimental Solid Mechanics and Dynamics,
School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran

¹hkorayem@iust.ac.ir; ²m.irani.r@gmail.com ; ³saerafee@yahoo.com

Abstract—In this paper, two methods are presented for solving closed loop optimal control problem and finding dynamic load carrying capacity (DLCC) for fixed and mobile manipulators. These control laws are based on the numerical solution to nonlinear Hamilton-Jacobi-Bellman (HJB) equation. First approach is the Successive Approximation (SA) for finding solution of HJB equation in the closed loop form and second approach is based on solving state-dependent Riccati equation (SDRE) that is an extension of algebraic Riccati equation for nonlinear systems. Afterward dynamic load carrying capacity of manipulators is computed using these controllers. The DLCC is calculated by considering tracking error and limits of torque's joints. Finally, results are presented for two cases, a two-link planar manipulator mounted on a differentially driven mobile base and a 6DOF articulated manipulator (6R). The simulation results are verified with the experimental test for the 6R manipulator. The simulation and experimental results demonstrate that these methods are convenient for finding nonlinear optimal control laws in state feedback form and finding the maximum allowable load on a given trajectory.

Index Terms—manipulator, DLCC, optimal feedback, SDRE, SA

I. INTRODUCTION

The DLCC of manipulators is one of the important characteristics of manipulators that restricted by limits of motor torques and tracking accuracy. In [1] a technique is introduced for computing the DLCC based on linear programming (LP) and then the DLCC is determined for specified path. In [2, 3] the DLCC problem is converted to an optimization problem and iterative linear programming (ILP) is used for solving this subject, and then this method is employed to find the DLCC of flexible manipulators and mobile robots. Korayem and Nikoobin [4] calculated maximum allowable load of mobile manipulator for a point-to-point motion by applying open loop optimal control approach.

Closed loop methods are used for determining Load carrying capacity, in [5] finding the DLCC of flexible joint robots is presented via the sliding mode control. Korayem et al [6] computed load carrying capacity of flexible joint manipulators using feedback linearization approach. Limitation on motor torques is one of factors that restricts the DLCC on a given trajectory, whatever the torque's motors are decreased, the DLCC will be increased, so using closed loop optimal control can increase the quantity of load capacity. Ravan and poulsen [7] presented analysis and design of

flexible manipulators using LQG controller for tracking of a pre-defined trajectory. Lee and Benli [8] designed an optimal control law for a flexible robot arm via linearization of equations.

In this paper, finding maximum dynamic load of manipulators is considered using closed loop nonlinear optimal control approach. For this purpose the nonlinear HJB equation, appeared in nonlinear optimal control problem must be solved. In general case this equation is a nonlinear partial differential equation that several methods are discussed for solving it [9].

In this paper, two applicable methods are considered for finding a solution to nonlinear HJB equation for fully nonlinear dynamics of manipulators. In the first method, numerical approximation based on Galerkin approach is used for solving HJB equation. The explanation and some applications of this method for solving optimal control problem are indicated in [10, 11]. In this method, designing procedure is implemented off-line and the execution time and convergence of algorithm is dependent on the selection of input parameters. In the second method, state-dependent Riccati equation technique is employed for finding optimal control law. This method is introduced in [12] and then is developed by Wernli and Cook [13] and Cloutier [14]. In addition, stability analysis of the method is considered in [15] by Hammet and Brett. This method is used to design controller for rigid and flexible manipulator [16, 17, 18 and 19]. The advantage of this method is that computing nonlinear optimal control takes place systematically.

Power series approximation is applied to solve the SDRE equation numerically. Finally the DLCC of mobile two-link robot and 6R manipulator is determined using these two controllers and then results for predefined trajectory are demonstrated and simulation results for 6R manipulator compared with the experimental test.

II. SOLUTION OF NONLINEAR OPTIMAL CONTROL PROBLEM

A. FIRST METHOD: SUCCESSIVE APPROXIMATION

For a system with the following general dynamics equation:

$$\dot{x} = f(x) + g(x)u(x) \quad (1)$$

With a cost function as:

$$J(x) = \int_0^{\infty} (l(x) + u(x)^T R u(x)) dt \quad (2)$$

The purpose is finding control law $u(x)$ to minimize the

cost function. In (2), $l(x)$ is positive definite that can be expressed by $x^T Q x$ where Q is a positive semi definite matrix; also, R is a positive definite matrix. It can be shown that the optimal control law is achieved via solving Hamilton-Jacobi-Bellman (HJB) equation, which has the following form:

$$\frac{\partial J^*}{\partial x} f(x) + l(x) - \frac{1}{2} \frac{\partial J^*}{\partial x} g(x) R^{-1} g^T(x) \frac{\partial J^*}{\partial x} = 0 \quad (3)$$

With finding the solution of (3), the optimal control law is achieved as follow:

$$u^*(x) = -\frac{1}{2} R^{-1} g^T(x) \frac{\partial J^*}{\partial x} \quad (4)$$

Equations (3) and (4) can be rewritten as follows:

$$\frac{\partial J^{(i)T}}{\partial x} [f + g u^{(i)}] + l + u^{(i)T} R u^{(i)} = 0 \quad (5)$$

$$u^{(i+1)} = -\frac{1}{2} R^{-1} g^T \frac{\partial J^{(i)}}{\partial x} \quad (6)$$

Equations (5) and (6) are solved by iteration method with an initial value $u^{(0)}$. Equation (5) is known as generalized Hamilton-Jacobi-Bellman (GHJB). The analytical solution of the GHJB is not possible and it is solved by numerical methods such as Galerkin method. According to the Galerkin method, the cost function $J^{(i)}(x)$ for each step can be expanded as a combination of specific basis functions, shown as $\{\phi_j(x)\}_{j=1}^{\infty}$. These basis functions are selected so that the state variables are continuous and bounded in reign Ω .

So the function $J^{(i)}(x)$ is rewritten as follow:

$$J^{(i)} = \sum_{j=1}^{\infty} C_j^{(i)} \phi_j(x) \quad (7)$$

An approximation of $J^{(i)}(x)$ for numerical solution is the finite number of basis functions as

$$\left\{ \phi_j(x) \right\}_{j=1}^N$$

where N is the order of approximation:

$$J_N^{(i)} = \sum_{j=1}^N C_j^{(i)} \phi_j(x) \quad (8)$$

Substituting the (8) in (5), the error of approximation can be written as:

$$e^{(i)}(x) = \frac{\partial \left(\sum_{j=1}^N C_j^{(i)} \phi_j \right)}{\partial x} (f + g u^{(i)}) + l + u^{(i)T} R u^{(i)} \quad (9)$$

According to the Galrkin method, the inner product of ϕ_j and the error function must be zero:

$$\langle e^{(i)}(x), \phi_j(x) \rangle_{\Omega} = 0 \quad j = 1, \dots, N \quad (10)$$

Where the inner product of two functions f and g is defined as follow:

$$\langle f(x), g(x) \rangle_{\Omega} = \int_{\Omega} f(x) g(x) dx \quad (11)$$

Coefficients vector $C^{(i)} = [C_1^{(i)}, C_2^{(i)}, \dots, C_N^{(i)}]^T$ is computed through solving (10) and then $u^{(i)}$ is achieved via (6).

The iteration process is repeated until the value of same coefficients become equal in during two steps with approximation accuracy.

It must be noted that the initial control vector $u^{(0)}$ is

selected so that the system be stable and all vectors $u^{(i)}$ computed in the next steps have the same property with the difference that control $u^{(i+1)}$ has better performance than $u^{(i)}$ [11].

B. SECOND METHOD: STATE-DEPENDENT RICCATI EQUATION

Using State-dependent Riccati equation technique to design nonlinear optimal control law, the function $f(x)$ in equation (1) must be parameterized as below:

$$f(x) = A(x)x \quad (12)$$

That the matrix $A(x)$ is nonlinear function of state variables. Then, SDRE equation is formulated as:

$$p(x)A(x) + A^T(x)p(x) - p(x)g(x)R^{-1}g^T(x)p(x) + Q = 0 \quad (13)$$

This equation must be solved for a positive definite state dependent matrix $p(x)$. The nonlinear optimal feedback law is established as:

$$u = -R^{-1}g^T(x)p(x)x \quad (14)$$

The challenge of the SDRE method is finding the solution of (13) that usually is difficult to get it analytically, so it can be numerically solved.

For solving the SDRE equation power series approximation is employed, for this purpose equations of system are rewritten as:

$$\dot{x} = A(x)x + B(x)u \quad (15)$$

$A(x)$ and $p(x)$ are rewritten as series:

$$A(x) = A_0 + \sum_{j=1}^m f_j(x) \Delta A_j \quad (16)$$

$$P(x) = L_0 + \sum_{j=1}^m f_j(x) L_j^j \quad (17)$$

Inserting $A(x)$ and $p(x)$ in (13), L_0 and L_j^j are computed using following equations [9]:

$$L_0 A_0 + A_0^T L_0 - L_0 B_0 R^{-1} B_0^T L_0 + Q = 0 \quad (18)$$

$$L_j^j (A_0 - B R^{-1} B^T L_0) + (A_0^T - B R^{-1} B^T L_0^T) L_j^j + L_0 \Delta A_j + \Delta A_j^T L_0 = 0 \quad (19)$$

$j = 1, 2, \dots, m$

Where (18) is the Algebraic Riccati equation and (19) are Lyapunov equations. Then the optimal control law is calculated via (14).

III. DYNAMIC MODELING OF MANIPULATORS

A. TWO-LINK MOBILE MANIPULATOR

Consider a two-link mobile manipulator with a schematic view shown in fig. 1.

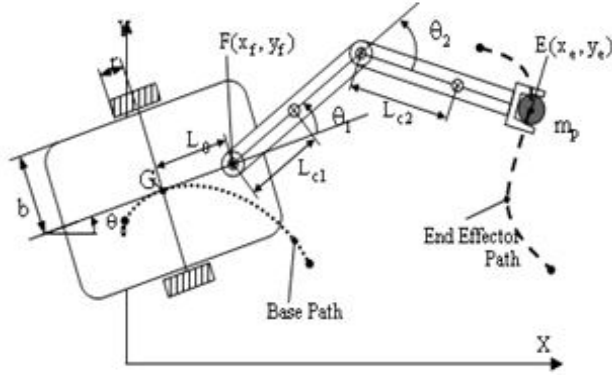


Figure 1. Mobile two-link manipulator.

Parameters of manipulator are shown in the table I [20].

TABLE I. PARAMETERS OF MOBILE MANIPULATOR

Parameters	Value	Unit
Length of links	$L2=L1=0.5$	m
Center of mass	$Lc2=Lc1=0.25$	m
Mass of links	$m1=5, m2=3$	Kg
Moment of inertia	$I1=0.416, I2=0.0625$	$Kg.m^2$
Mass of wheel	5	Kg
Mass of base	94	Kg
Moment of inertia of base	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6.609 \end{bmatrix}$	$Kg.m^2$
Moment of inertia of wheels	$\begin{bmatrix} 0.131 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.131 \end{bmatrix}$	$Kg.m^2$
b	0.171	m
r	0.075	m
$L0$	0.4	m

As described in [20], Dynamic equations of motion are obtained using Lagrange method:

$$\begin{bmatrix} F_x \\ F_y \\ T_0 \\ \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{12} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{13} & J_{23} & J_{33} & J_{34} & J_{35} \\ J_{14} & J_{24} & J_{34} & J_{44} & J_{45} \\ J_{15} & J_{25} & J_{35} & J_{45} & J_{55} \end{bmatrix} \begin{bmatrix} \ddot{x}_f \\ \ddot{y}_f \\ \ddot{\theta}_0 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix} \quad (20)$$

Parameters in (20) are presented in [20]. The degree of freedom of end effector is two, and the degree of freedom of system implied by (20) is five, thus the order of redundancy of

the system is three that is the number of required constraints, which must be applied for redundancy resolution. The number of nonholonomic constraints of the system is one and it is the form of [20]:

$$\dot{x}_f \sin(\theta_0) - \dot{y}_f \cos(\theta_0) + L_0 \dot{\theta}_0 = 0 \quad (21)$$

Two holonomic constraints that apply to system are the predefined path for the base coordinates x_f, y_f . Then $\ddot{x}_f, \ddot{y}_f, \dot{x}_f, \dot{y}_f$ are gained and $\theta_0, \dot{\theta}_0, \ddot{\theta}_0$ are achieved from (21). With these assumptions, the equations of the system (20) can be rewritten as follow:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} J_{44} & J_{45} \\ J_{45} & J_{55} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \quad (22)$$

Where:

$$\begin{aligned} R_1 &= J_{14}\ddot{x}_f + J_{24}\ddot{y}_f + J_{34}\ddot{\theta}_0 + C_4 \\ R_2 &= J_{15}\ddot{x}_f + J_{25}\ddot{y}_f + J_{35}\ddot{\theta}_0 + C_5 \end{aligned} \quad (23)$$

For finding state space equations, state variables are chosen as:

$$X_1 = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, X_2 = \begin{bmatrix} \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} x_3(t) \\ x_4(t) \end{bmatrix} \quad (24)$$

Then state space representation is:

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = P(J_{33}(U_1 - R_1) - J_{35}(U_2 - R_2)) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = P(-J_{45}(U_1 - R_1) + J_{44}(U_2 - R_2)) \end{cases} \quad (25)$$

Where

$$P = 1/(J_{44}J_{55} - J_{45}^2)$$

B. 6R FIXED ROBOT

Consider a six degree of freedom robot that is shown in Fig. 2[21].

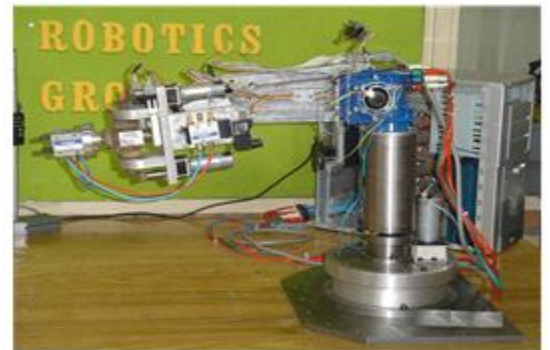


Fig 2. 6R robot

The dynamic equation of motion of this robot can be obtained as:

$$\tau(t) = D(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t)) + G(q(t)) \quad (26)$$

q is the vector of angular position of joints:

$$q = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5 \ \theta_6] \quad (27)$$

Angular positions and velocities of links are selected as state variables; therefore, the state-space representation is obtained as:

$$\dot{X} = [x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, D^{-1}(U - C - G)] \quad (28)$$

In (28), D , C and G are the inertial matrix, the vector of Coriolis and centrifugal forces and the gravity force vector, respectively. U in this equation is the input control vector.

IV. SIMULATION AND EXPERIMENTAL RESULTS

A. CASE STUDY I

The function $l(x)$ in (2) is considered as:

$$l(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 \quad (29)$$

And matrix R is selected $I_{2 \times 2}$ for both methods. Basis functions required for the Galerkin algorithm are selected as:

$$\{\phi_j\}_{j=1}^{10} = \{x_1^2, x_1x_2, x_2^2, x_1x_3, x_2x_3, x_3^2, x_1x_4, x_2x_4, x_3x_4, x_4^2\} \quad (30)$$

Limits of state variables are:

$$\begin{aligned} -3\pi \text{ rad} &\leq x_1 \leq +3\pi \text{ rad} \\ -4 \text{ rad/sec} &\leq x_2 \leq +4 \text{ rad/sec} \\ 0.1 \text{ rad} &\leq x_3 \leq 3 \text{ rad} \\ -4 \text{ rad/sec} &\leq x_4 \leq +4 \text{ rad/sec} \end{aligned} \quad (31)$$

An initial control vector $u^{(0)}(x)$ required in SA method is design using LQ technique:

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} -3.16, 0, -7.65, -1.09 \\ 0, -3.16, -1.09, -3.67 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} \quad (32)$$

The upper and lower bounds of torque of motors are as follows:

$$\begin{aligned} u_1^+ &= k_1 - k_2 \dot{q} \\ u_2^- &= -k_1 - k_2 \dot{q} \end{aligned} \quad (33)$$

Where $k_1 = \tau_s$, $k_2 = \tau_s / w_{nl}$, also τ_s is stall torque and

w_{nl} is no load speed of motor.

Two controllers based on SA and SDRE methods are designed for two-link mobile manipulator to track a square trajectory with 2cm allowable tracking error. Trajectories using two methods and related variations of tracking error are presented in Fig. 3.

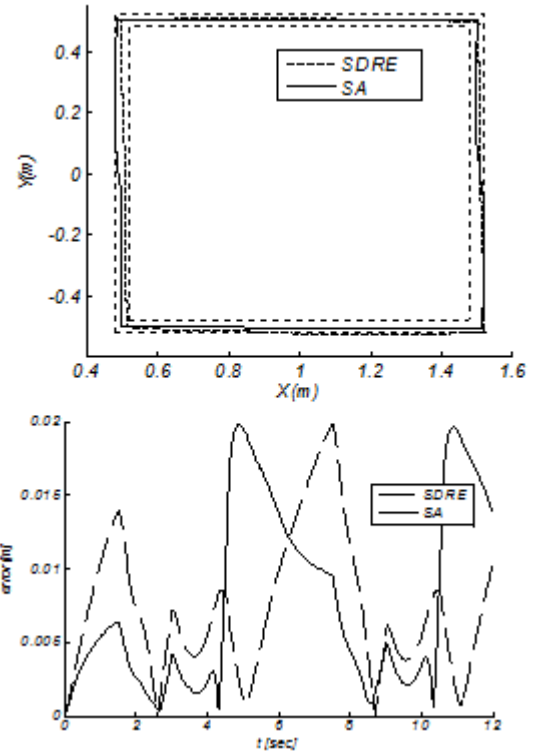


Fig 3. End effector trajectory and Tracking error.

Dynamic load carrying capacity of manipulator is obtained 5 kg using controller based on SA method and 5.9 kg for SDRE controller. The results demonstrate that the DLCC obtained using SDRE controller is higher than using SA controller. Necessary torques of joints for trajectory tracking is shown in Fig. 4.

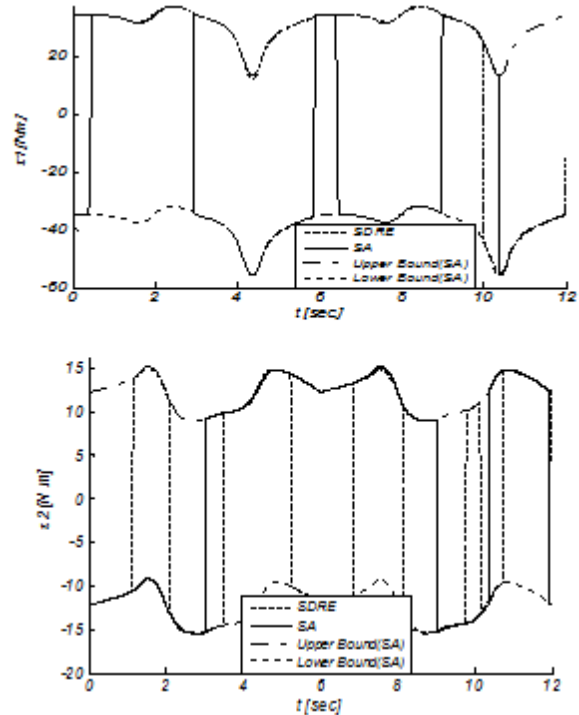


Fig 4. Motor torque of links.

A. CASE STUDY 2

As second study, a circular trajectory is selected for tracking problem of 6R robot. The initial angular positions of links are:

$$\theta_{init} = [-0.98 \ -0.38 \ 0.63 \ 0.7 \ -\pi/2 \ -\pi/2]^T \quad (34)$$

Equations of desired the circular trajectory that must be happened in 2π seconds are:

$$\begin{aligned} x_d &= 0.2 \cos(t) + 0.1 \\ y_d &= 0.05 \sin(t) - 0.45 \\ z_d &= 0.1 \cos(t) + 0.3 \end{aligned} \quad (35)$$

Because the second method reaches better DLCC and it can be implemented systematically, a nonlinear optimal controller is designed using SDRE method for simulation study.

The function $l(x)$ in (2) is considered as standard form $x^T Q x$

where $Q = I_{12 \times 12}$ and matrix R is selected to be $I_{6 \times 6}$.

For an allowable tracking error equal to 2cm, the DLCC of robot is computed 0.9 kg. The value of the DLCC is a function of matrixes Q and R , the value of admissible error and the characteristics of motors as (33).

Fig. 5 shows three different trajectories for end effector: desired trajectory, trajectories obtained in simulation and experimental test.

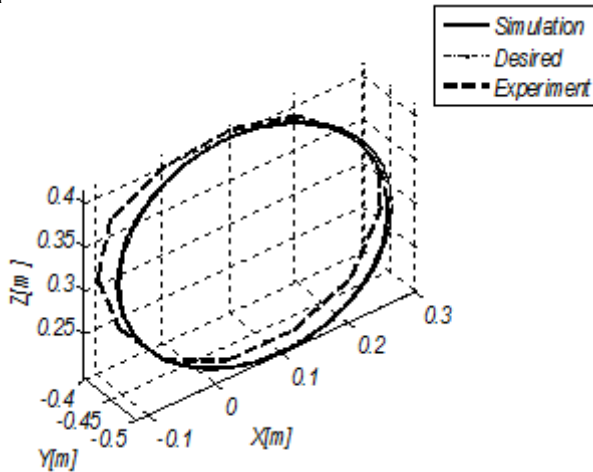


Fig 5. End effector path during tracking.

Figure (6a) presents simulation results for angular positions of joints in full load conditions. This figure indicates smooth angular motion for joints during the motion. Figure (6b) shows experimental test results of angular position of links.

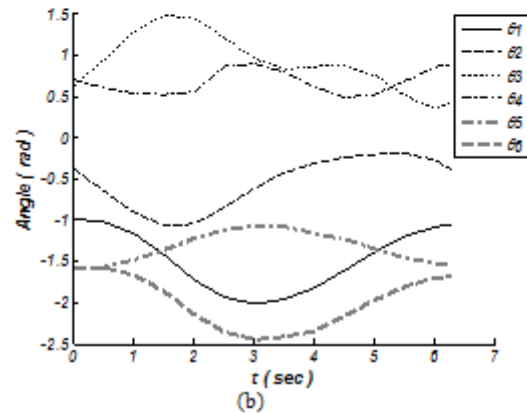
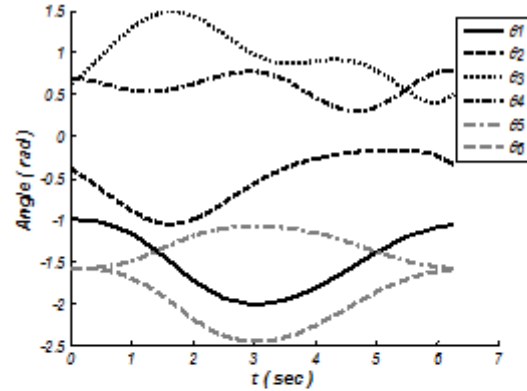


Fig 6. Angular position of joints: a. simulation, b. experimental test.

CONCLUSION

In this paper, two nonlinear methods are applied for designing nonlinear optimal controller of both fixed and mobile manipulators and determining dynamic load carrying capacity of them that is an important characteristic of a manipulator. The results indicate that the system response is appropriate and acceptable. It should be noted that the structure of controller is nonlinear feedback of the state variables, in other words it is a closed loop control system. It should be noted that in Successive Approximation method, various selections of Q and different number and type of basis functions will result different controllers with different properties that is one of the advantages of this method. Determining the basis functions is the main point of method because that the convergence of Galerkin algorithm is depend

on the type of basis functions and the number of these functions determines the required memory and speed of numerical calculations. A usual way for selecting basis functions is try and error procedure. Another point that should be mentioned is that the calculations of SA method are done automatically and Off-Line by a computer. However State Dependent Riccati Equation method provides a systematic way to deal with nonlinear control systems. Extra design degrees of freedom arising from the various parameterizations in the form of (14) also the performance of designed controller is related on the selection of R and Q matrixes.

REFERENCES

- [1] L.T. Wang and B. Ravani, "Dynamic Load Carrying Capacity of Mechanical Manipulators-Part 1", J. of Dynamic Sys. Measurement and Control, vol. 110, pp. 46-52, 1988.
- [2] H. Ghariblu and M. H. Korayem, "Trajectory Optimization of Flexible Mobile Manipulators", Robotica, vol. 24, pp. 333-335, 2006.
- [3] M.H. Korayem, H. Ghariblu and A. Basu, "Maximum Allowable Load of Mobile Manipulator for Two Given End Points of End-Effector", Int. J. Of AMT, vol. 24, pp. 743 – 751, 2004.
- [4] M.H. Korayem and A. Nikoobin, "Maximum Payload for Flexible Joint Manipulators In Point-To-Point Task Using Optimal Control Approach", Int. J. Of AMT, vol. 38, pp. 1045-1060, 2007.
- [5] H. Korayem and A. Pilechian, "Maximum Dynamic Load Carrying Capacity In Flexible Joint Robots Using Sliding Mode Control", Int. Congress On Manufacturing Engineering, Tehran, Iran, December 2005.
- [6] M. H. Korayem, F. Davarpanah and H. Ghariblu, "Load Carrying Capacity of Flexible Joint Manipulator with Feedback Linearization", Int J Adv Manuf Technol., vol. 29, pp. 389-397, 2006.
- [7] O. Ravn and N.K. Poulsen, "Analysis and Design Environment for Flexible Manipulators", Chapter 19 In Tokhi, M.O. And Azad, A.K.M.: Flexible Robot Manipulators – Modeling, Simulation and Control, 2004.
- [8] J Lee, Benli Wang, "Optimal Control of a Flexible Robot Arm", Computers and Structures, vol.29, No.3, pp. 459-467, 1988.
- [9] S. C. Beeler, H.T. Tran and H.T. Banks, "Feedback Control Methodologies for Nonlinear Systems", Journal Of Optimization Theory And Applications, vol. 107, pp. 1-33, 2000.
- [10] R.W. Beard and T.W. McLain, "Successive Galerkin Approximation Algorithms for Nonlinear Optimal and Robust Control", International Journal of Control: Special Issue On Breakthroughs In The Control Of Nonlinear Systems, vol. 71, No.5, pp. 717-743, 1998.
- [11] T.W. McLain and R.W. Beard, "Successive Galerkin approximations to the nonlinear optimal control of an underwater robotic vehicle," in Proceedings of the 1998 International Conference on Robotics and Automation, pp. 762–767, Leuven, Belgium, May 1998.
- [12] J.D. Pearson, "Approximation Methods in Optimal Control" I. Suboptimal Control, J. Electronics and Control 13, pp. 453-469, 1962.
- [13] A. Wernli, And G. Cook, "Suboptimal Control for the Nonlinear Quadratic Regulator Problem" Automatica, vol. 11, pp. 75-84, 1975.
- [14] J. R. Cloutier, "State-Dependent Riccati Equation Techniques an Overview" American Control Conference, pp. 932-936, Albuquerque USA, Jun 1997.
- [15] K. Hammet and D. Brett, "Relaxed Conditions for Asymptotic Stability of Nonlinear SDRE Regulators" 36th IEEE Conference on Decision and Control, pp. 4012-4017, San Diego USA, Dec 1997.
- [16] E.B. Erdem, A.G. Alleyne, "Experimental Real-Time SDRE Control of an under actuated Robot" The 40th IEEE Conference on Design and Control, pp. 2986-2991, Florida, Dec 2001.
- [17] M. Innocenti, F. Baralli and F. Salotti, "Manipulator Path Control Using SDRE" Proceedings of the American Control Conference, pp. 3348-3352, Chicago, Illinois June, 2000.
- [18] M. Xin, S.N. Balakrishnan and Z. Huang, "Robust State Dependent Riccati Equation Based Robot Manipulator Control" The Conference on Control Applications, pp. 369-374, September, 2001.
- [19] A. Shawky, A. Ordys and M.J. Gremble, "End-Point Control Of A Flexible-Link Manipulator Using Hinf Nonlinear Control Via State-Dependent Riccati Equation" The Conference On Control Applications, pp. 501-506, September 2002.
- [20] M.H. Korayem, A. Nikoobin and V. Azimirad, "Maximum load carrying capacity of mobile manipulators: optimal control approach", Robotica, vol. 27, pp. 147-159, 2009.
- [21] M.H. Korayem and F.S. Heidari, "Simulation and experiments for a vision-based control of a 6R robot", J. of AMT, vol. 41, No. 4, pp. 367-385, 2008.